One dimensional case­:

Closed form LQR

In this case and

Using Variational Calculus optimization formula.

We’ll optimize the following function:

Let’s see that the values of remains constant in the values of the optimum solutions:

We also know:

So the optimal policy for a one dimensional system is:

Substituting in the dynamical system:

To prove optimality, it’s enough to satisfy the HJB Equation:

Choosing as:

Choosing as

The following condition must hold

Replacing back this equation, we get: